noise ratio are the down-conversion sensitivity of the oscillator (T_1), the signal level (V_0), and the RF noise voltage ($\overline{e_n^2}$), the first two of which should be large (within the limitations of the small-signal noise analysis [20]) and the third, small. The voltage sensitivity of the device impedance does not influence the signal-to-noise ratio because it influences both the Doppler signal power, (12) and (14), and AM noise power, (28), equally. The calculated signal-to-noise ratio for a self-mixing ATT diode oscillator used in a Doppler radar shows that for a given diode it is desirable to operate it at a high frequency and large power output. Alternatively, for a given radar the signal-to-noise ratio is improved by choosing a diode with a longer depletion-region length. $1/f$ noise in the device, if any, was neglected in this analysis; the method of calculation is, however, general.

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Short Papers

Scattering Parameter Approach to the Design of Narrow-Band Amplifiers Employing Conditionally Stable Active Elements

C. S. GLEDHILL AND M. F. ABULELA

Abstract—**In terms of scattering parameters, the equation of transducer power gain is shown to be capable of representation as a family of circles of constant gain from which the design of load and source terminations to achieve a restricted bandwidth can be obtained. This is an extension of an earlier approach which only allowed either load reflection coefficient or source reflection coefficient to be considered in a given design. Through the use of a specification statement of VSWR, it is shown how a marginal stability factor can be derived. From the study of the interaction between the input and output reflection coefficients, a detuning factor is analytically derived to correlate the interaction between the input and output reflection coefficients. Either of these factors can be chosen and used to select optimum input and output reflection coefficients which provide stable operation for an amplifying stage that is to employ a conditionally stable active element. An example using these factors is given.**

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I. Introduction

Over the past few years, there has been an increasing interest in the transistor two-port scattering parameters [1]–[4] further extended to linear integrated circuits [5]. Their use in amplifier design has been formalized [6], [7], and in particular Bodway [7] has carried out a full investigation from a power-gain point of view into the unconditionally stable case of the active two-port. Under normal circumstances, each stage of an amplifier is to be operated under the dc bias conditions which will provide the highest value of maximum unconditionally stable transducer power gain [7]. For an active element that shows a conditionally stable (or unstable) quiescent operating point, the logical solution is to look for another dc one which shows unconditional stability.

However, situations may arise when one must use the active element under predetermined dc quiescent values imposed by a higher priority criterion, e.g., noise figure in a front-end stage. Such a case may yield ac parameters showing conditional stability, and while the unconditionally stable case provides unique values of source and load reflection coefficients at the maximum transducer power gain, the conditionally stable case does not. This is why it is necessary to look for a new criterion on which to base the termination selection. Such a criterion will therefore be required: 1) to provide complete stabilization for a conditionally stable case; 2) to be analytically related to the characterization of the active element; 3) to be related to the specification requirements.

In this short paper, the case of the conditionally stable two-port is considered. Starting with a reexamination of the transducer power-gain equation, followed by a discussion of the stabilization approaches, the design formulas incorporating the stabilization factors and bandwidth requirements are given in terms of the scattering parameters.

Relationship Between Gain and Stability in Terms of Scattering Parameters

The definitions of gain have been formulated as follows [7]:

$$\text{power gain} = G = \frac{\text{power delivered to load}}{\text{power into two-port}}$$

$$= \frac{|S_{21}|^2 (1 - |r_2|^2)}{|(1 - |S_{11}|^2) + |r_2|^2(|S_{22}|^2 - |\Delta|^2) - 2\,\mathrm{Re}\,(r_2C_2)|} \tag{1}$$

$$\text{available power gain} = G_A = \frac{\text{power available at the output}}{\text{power available from the generator}}$$

$$= \frac{|S_{21}|^2 (1 - |r_1|^2)}{|(1 - |S_{22}|^2) + |r_1|^2(|S_{11}|^2 - |\Delta|^2) - \mathrm{Re}(r_1C_1)|} \tag{2}$$

$$\text{transducer power gain} = G_T = \frac{\text{power delivered to load}}{\text{power available from generator}}$$

$$= \frac{|S_{21}|^2 (1 - |r_1|^2)(1 - |r_2|^2)}{|1 - r_1S_{11} - r_2S_{22} + r_1r_2\Delta|^2} \tag{3}$$

in which

$$r_i = \frac{Z_i - Z_{i0}}{Z_i + Z_{i0}^*} \tag{4}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \tag{5}$$

$$C_1 = S_{11} - S_{22}^*\Delta \tag{6}$$

$$C_2 = S_{22} - S_{11}^*\Delta. \tag{7}$$

Re indicates real part of and * indicates conjugate. Further, the stable and unstable regions of operation have been shown to be defined in the r_i plane circles of the general form:

$$\left| r_i - \frac{C_i^*}{|S_{ii}|^2 - |\Delta|^2} \right|^2 = \left| \frac{S_{12}S_{21}}{|S_{ii}|^2 - |\Delta|^2} \right|^2 \tag{8}$$

or

$$|r_i - r_{si}|^2 = |\rho_{si}|^2. \tag{9}$$

Stable operation occurs outside the ρ_{si} circle, shown in Fig. 1, and since for physically realizable two-port source and load reflection coefficients, $|r_i|$ must be less than unity, it follows that unconditionally stable operation is possible if the ρ_{si} circle lies completely outside the unity circle in the r_i plane, and the origins are stable, i.e.,

$$|\rho_{si} - |r_{si}|| > 1 \quad \text{and} \quad |s_{ii}| < 1.$$

Under these conditions simultaneous conjugate matching of both ports is possible, and the maximum transducer gain is achieved [7]. Alternatively, the situation can occur where the ρ_{si} circle is fully or partially included in the unity circle, thus yielding a case of conditional stability.

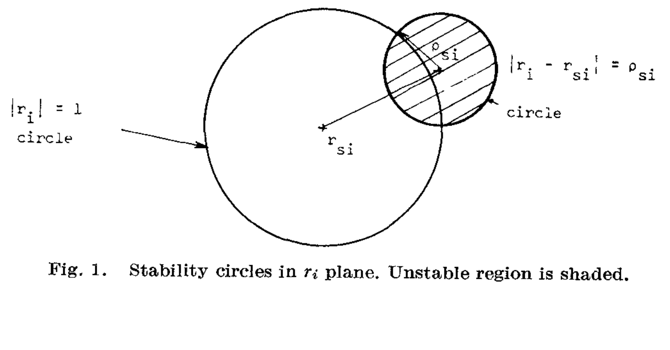

Fig. 1. Stability circles in r_i plane. Unstable region is shaded.

II. Constant Power Gain and Stability Circles

From (1)–(3) it can be seen that for the design of the load and source impedances, first r_2 can be optimized from consideration of the equation of power gain (1) and then r_1 can be presented as families of circles of constant power gains. From (1)

$$\frac{1}{g_2} = -D_2 + \left[\frac{B_2 - 2\,\mathrm{Re}\,(r_2C_2)}{1 - |r_2|^2}\right] \tag{10}$$

in which

$$g_2 = \frac{G}{|S_{21}|^2} \tag{11}$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \tag{12}$$

$$D_2 = |S_{22}|^2 - |\Delta|^2. \tag{13}$$

Rearranging (10),

$$\left| r_2 - \frac{g_2}{1 + D_2g_2} C_2^* \right|^2 = \frac{|(1 - 2K|S_{12}S_{21}|g_2 + |S_{12}S_{21}|^2 g_2^2)|^2}{|1 + D_2g_2|^2} \tag{14}$$

in which

$$K = \frac{(1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2)}{2|S_{12}S_{21}|} \tag{15}$$

(which has been defined as the invariant stability factor [9]). Equation (14) represents a family of circles defining load reflection coefficients giving the same power gain.

For physically realizable loads, $|r_2| \leq 1$ or $(1 - |r_2|^2) \geq 0$, and g_2 becomes infinite if

$$D_2(1 - |r_2|^2) = B_2 - 2\,\mathrm{Re}\,(r_2C_2). \tag{16}$$

This condition can be rewritten as

$$\left| r_2 - \frac{C_2^*}{|S_{22}|^2 - |\Delta|^2} \right|^2 = \left| \frac{|S_{12}S_{21}|}{|S_{22}|^2 - |\Delta|^2} \right|^2 \tag{17}$$

i.e., the same equation as (8), which defined the general stability condition.

Alternatively, families of circles defining source reflection coefficients giving the same available power gain could have been obtained, starting from (2). However, an equation relating both input and output reflection coefficients to the efficiency of power exchange can only be obtained from the transducer power-gain equation.

Starting from (3) for the transducer power gain

$$|r_1 - R_{OT1}|^2 = |\rho_{OT1}|^2 \tag{18}$$

in which

$$R_{OT1} = \frac{(1 - r_2S_{22})(S_{11}^* - r_2^*\Delta^*)}{\{|S_{11} - r_2\Delta|^2 + (1 - |r_2|^2)/g_t\}} \tag{19}$$

$$\rho_{OT1} = \left| \frac{\{[|S_{11} - r_2\Delta|^2 - |1 - r_2S_{22}|^2][(1 - |r_2|^2)/g_t] + [(1 - |r_2|^2)/g_t]^2\}^{1/2}}{|S_{11} - r_2|^2 + (1 - |r_2|^2)/g_t} \right| \tag{20}$$

$$g_T = \frac{G_T}{|S_{21}|^2}. \tag{21}$$

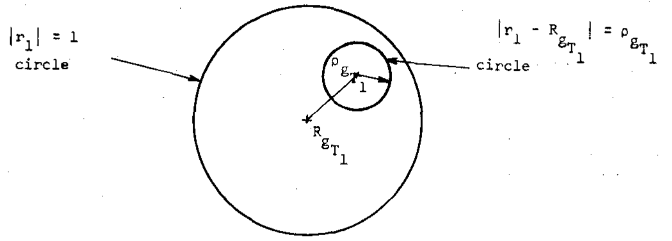

Fig. 2. Constant transducer power-gain circle in r_1 plane.

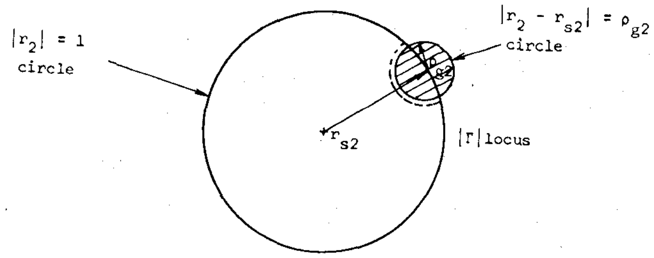

Fig. 3. Stabilization in r_2 plane.

From this equation, the circles shown in Fig. 2 can be drawn. Given an output termination and a specified transducer power gain, the input reflection coefficient, and hence the source impedance, can be calculated or can be obtained graphically. On the other hand, (14) only relates the output reflection coefficient to the power gain which, by definition, does not include the source conditions.

III. Stabilization Approach

Since it is always true that satisfactory stable operation will only be possible outside the stability circle (g_i equals ∞ circle) and inside the unit circle, there are two possible ways to achieve stable operation, namely:

a) to alter the network so that the stability circle lies completely outside the unit circle;

b) to limit the stage gain to a certain value, determined as follows.

A. Load Reflection Coefficient

Solution a), preceding, implies that one deliberately makes

$$\left| \frac{|C_2| - |S_{12}S_{21}|}{D_2} \right| > 1 \tag{22}$$

or

$$\{[(1 - |S_{11}|^2)(|S_{22}|^2 - |\Delta|^2) + |S_{12}S_{21}|]^{1/2} - |S_{12}S_{21}|\} > ||S_{22}^2| - |\Delta|^2|. \tag{23}$$

This is only possible by feedback, and the design equations are better derived from a three-port parameter approach [10].

For the second proposed solution, b), preceding, two possibilities exist.

i)

$$|r_2| \leq |\Gamma| \tag{24}$$

in which

$$|\Gamma| = M \, ||r_{S2}| - \rho_{S2}| \tag{25}$$

$$M = \frac{\nu(1 + ||r_{S2}| - \rho_{S2}|^{-1} - 0.5}{1 - \nu(1 + ||r_{S2}| - \rho_{S2}|)} \tag{26}$$

and $\nu = 1 - (\text{min VSWR})/(\text{max VSWR})$. M is called the marginal stability factor, determined by the maximum tolerable deviation (ν) of VSWR. This particular value of $|r_2| = |\Gamma|$ will ensure that the two-port is stable irrespective of the sign of D_2 [7]. In fact, this is equivalent to the approach to the loading of unstable amplifiers by Stern [8].

ii)

$$|r_2| \leq |\Gamma| \leq |P \pm (P^2 - Q)^{1/2}| \tag{27}$$

(this is derived in Appendix II), in which

$$P = \frac{\Delta + S_{11}S_{22} + \gamma S_{12}S_{21}}{2 . S_{22} . \Delta} \tag{28}$$

$$Q = \frac{S_{11}}{S_{22}\Delta} \tag{29}$$

and

$$\frac{1}{\gamma} = \delta_{r2} \text{ (reflection detuning factor)}. \tag{30}$$

In this, the reflection detuning factor is defined as the ratio of the fractional change in the two-port input reflection coefficient to the fractional change in the output load reflection coefficient, and the value of $|r_2|$ given by (27) is dependent on the maximum tolerable value of this that the design sensitivity requirement may impose. From (1) and (11) it is evident that G_2 will be a maximum when $2 \text{ Re } (r_2C_2)$ is a maximum, i.e., when the phase angle of r_2 is equal to the phase angle of C_2^*. Therefore, the optimum load reflection coefficient is $r_2 = |\Gamma| \arg C_2^*$ which is identical to

$$r_2 = |\Gamma| \arg \frac{C_2^*}{D_2} \tag{31}$$

which, from (11) shows that r_2 will have the same phase angle as r_{S2} (see Fig. 3). Note [7] that if D_2 is negative, the region of stable operation is inside the gain equals ∞ circle which also includes the origin, and therefore the conditions of (26), (27), and (31) still apply.

It will now be shown from consideration of the load quality factor Q that the optimum value of phase angle implicit in (31) may not always be obtainable.

The relationship between Q and r_2 is given by [11]

$$Q = \left| \frac{r_2(\omega_0) - r_2^*(\omega_0)}{1 - r_2(\omega_0)^2} \right| \tag{32}$$

giving

$$Q = \frac{1}{2} \frac{|\Gamma| \sin \phi_r}{1 - |\Gamma|^2}. \tag{33}$$

Hence

$$\phi_r = \sin^{-1}\left[2Q \frac{1}{|\Gamma|} - |\Gamma|\right]. \tag{34}$$

If $\Delta\omega_s$ and $\Delta\omega_p$ are defined, respectively, to be the desired stage bandwidth and the obtainable bandwidth, since $Q = (\omega_0)/(\Delta\omega_p)$, if $\Delta\omega_s \leq \Delta\omega_p$,

$$\phi_r = \arg\left(\frac{C_2^*}{D_2}\right). \tag{35}$$

But if $\Delta\omega_s > \Delta\omega_p$, the required phase angle cannot be obtained, the nearest approach to it being given from (34) by

$$\phi_r = \sin^{-1}\left[\frac{2\omega_0}{\Delta\omega_p}\left(\frac{1}{|\Gamma|} - |\Gamma|\right)\right] \tag{36}$$

in which, from (32),

$$\Delta\omega_p = \frac{\omega_0(1 - |\Gamma|^2)}{||\Gamma| \arg (C_2^*/D_2) - |\Gamma| \arg (C_2/D_2)|}. \tag{37}$$

B. Source Reflection Coefficient

The normalized transducer power gain g_T can be rewritten as

$$g_T = \frac{G_T}{|S_{21}|^2} = \frac{a(1 - |r_1|^2)}{|b + r_1 d|^2} \tag{38}$$

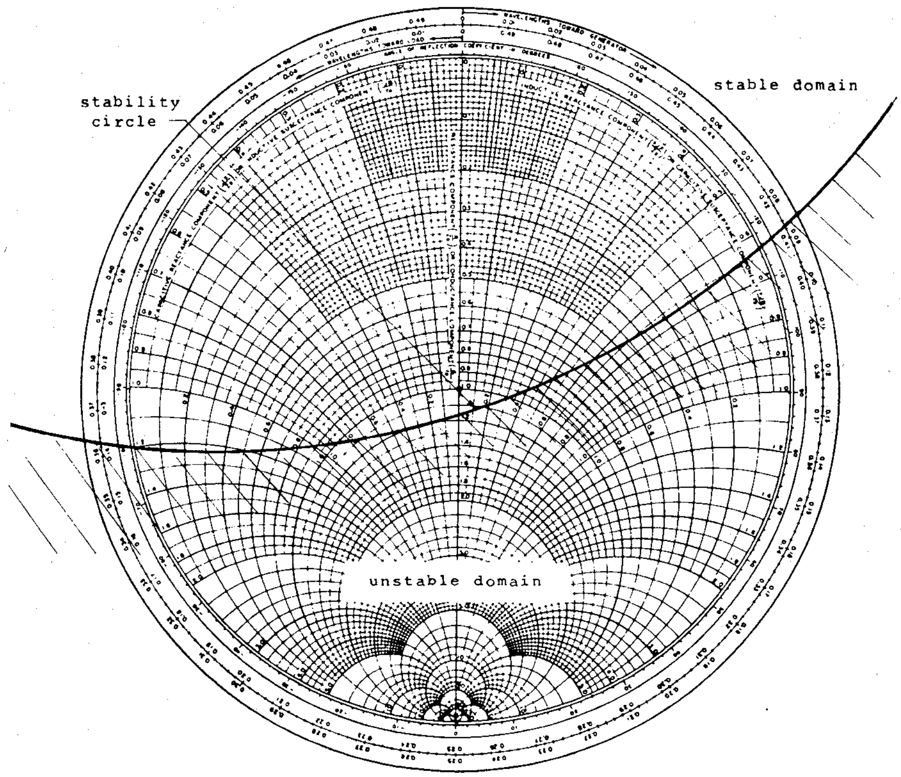

Fig. 4. r_1 plane.

in which

$$a = 1 - |r_2|^2 \quad (39)$$

$$b = 1 - r_2 S_{22} \quad (40)$$

$$d = r_2 \Delta - S_{11}. \quad (41)$$

If (38) was subject to restraints viz.

$$|r_1| \leq 1 \quad (42)$$

and

$$g_T \leq m \quad (43)$$

in which m is some value determined from consideration of the r_2 plane, then this value can be used to define the gain circle in (18) and the locus of permissible reflection coefficients found. Also, using the simultaneous condition imposed by the stage bandwidth $\Delta\omega_s$, there is obtained

$$\frac{\omega_0}{\Delta\omega_s} = \frac{|r_1(\omega_0) - r_1^*(\omega_0)|}{1 - |r_1(\omega_0)|^2} \quad (44)$$

and the particular circle

$$|r_1 - R_{g_T=m}| = \rho_{g_T=m}. \quad (45)$$

The solution of these last two equations fully defines and determines the magnitude and phase of r_1. In general, there will be two values, but realizability conditions allow only the value $|r_1| \leq 1$.

IV. Example

Defining a narrow-band amplifier as one having a bandwidth <10 percent of the midband frequency, and also one for which the linear approximations of the behavior of the physical components with respect to frequency hold, a typical example was designed and built [10]. A transistor, type BF 180, operated at $I_C = 1.0$ mA and $V_{CE} = 11$ V was designed to give the maximum gain over a $2\frac{1}{2}$-percent bandwidth at 800 MHz. The measured scattering parameters, with reference impedance of 50, were

$$S_{11} = 0.233\angle 106.04^\circ \qquad S_{12} = 0.163\angle 86.17^\circ$$

$$S_{21} = 1.032\angle -135.23^\circ \qquad S_{22} = 0.989\angle -32.94^\circ$$

from which it can be seen that the device is not potentially unstable.

Using (8) and (9) stability circles were drawn, as shown in Figs. 4 and 5, from which it can be seen that the device is conditionally stable at both input and output.

Taking the inherent minimum VSWR of the amplifier including connectors to be 1.1:1, and specifying the maximum VSWR to be 1.13:1, we obtained, from (26), the marginal stability factor to be zero, which in turn determined the input reflection coefficient r_1 to be at the origin of the r_1 plane.

From (27) r_2 was derived and shown in Fig. 5 as two curves for various values of r_2 between 0.01 and 1.0. From these r_2 was chosen to be 0.2, giving $r_2 = 0.824\angle 37.53^\circ$. From (3) the transducer power gain, G_T, was found to be 9.39 dB.

Using the methods of [11], the design was finalized as shown in Fig. 6 with the measured response shown in Fig. 7. It should be noted that the expression for Q, [11, eq. (1)], was derived in the Appendix to that paper by a method which was not generally true, and the following derivation is preferred.

Assume that the driving-point impedance at a given port is given by $R + jX$, which when normalized to Z_0 is $r + jx$. Then, $\Gamma = (r + jx - 1)/(r + jx + 1)$ and $\Gamma^* = (r - jx - 1)/(r - jx + 1)$. Hence by simple manipulation

$$\frac{|\Gamma - \Gamma^*|}{|1 - |\Gamma|^2} = \frac{x}{r} = Q.$$

Of course, considered at the particular frequency at which $x = 0$, Q apparently equals zero, which is precisely why the alternative form $Q(\omega) = \omega/2\Delta\omega$ has been derived as an approximate formula for engineering design. The derivation, however, employs the formula $Q = x/r$.

Appendix I

Marginal stability factor based on the tolerable deviation of VSWR. If $|\Gamma'|$ is the radius of the circle, centered at the origin, which is tangential to the g equals ∞ circle, then the corresponding

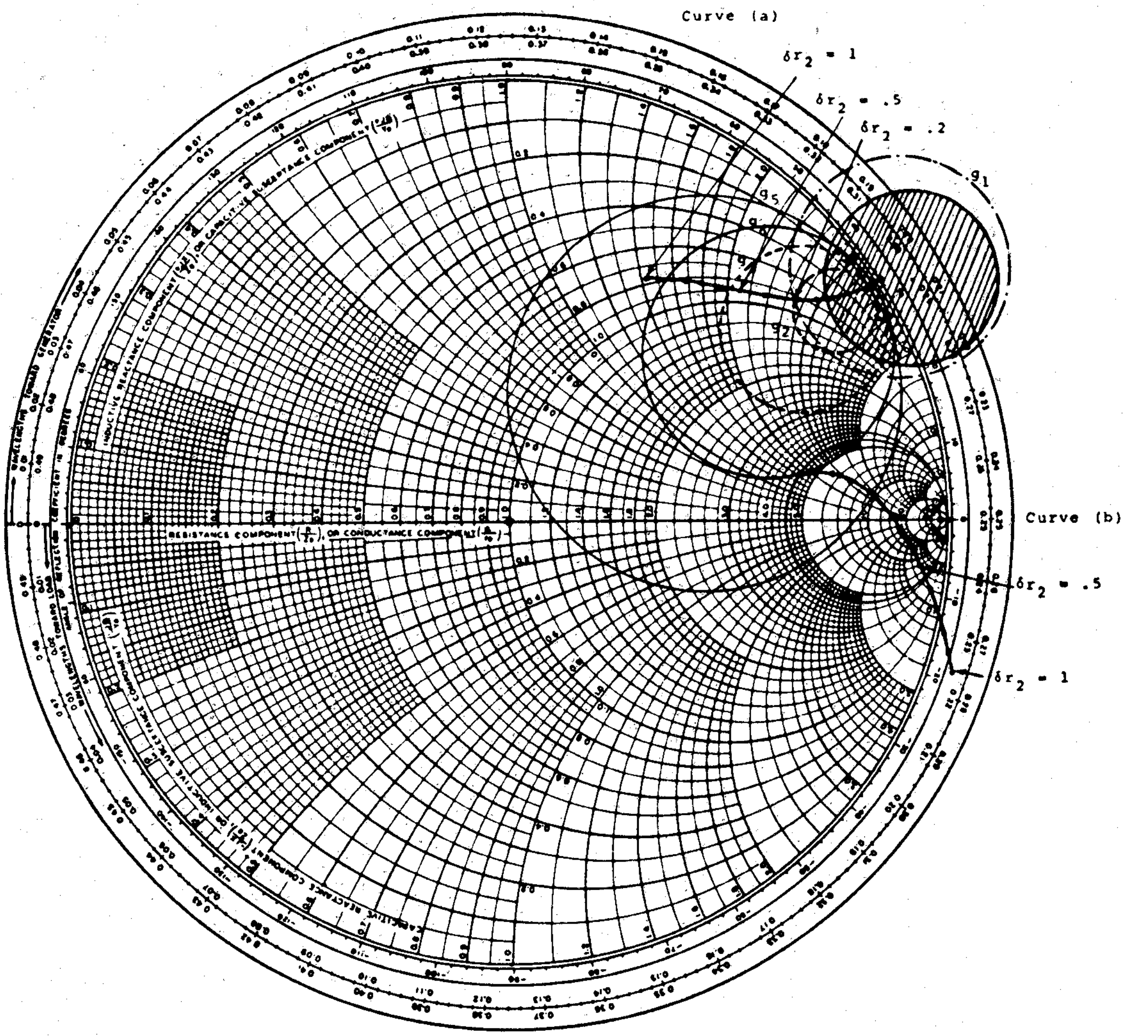

Fig. 5. Stability and gain circles in r_2 plane. Constant transducer power-gain circles q. $q_2 = 9.4$ dB; $q_3 = 6.4$ dB; $q_4 = 4.1$ dB; $q_5 = 1.1$ dB. All for $r_1 = 0$.

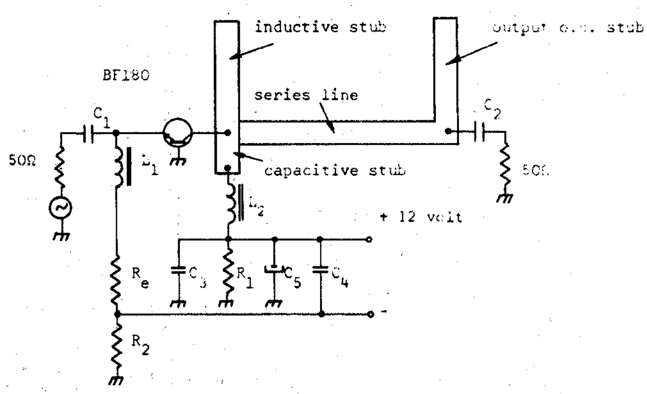

Fig. 6. Circuit diagram of stabilized 800-MHz narrow-band single-stage amplifier designed using two-port S-parameters. $v_{CC} = +12$ V; $R_1 = 18$ kΩ, $R_2 = 3.9$ kΩ, $R_e = 1$ kΩ; $L_1 = L_2 = 0.15$ μH (RF choke); $C_1 = 0.1$ μF disk ceramic; C_2 200 pF ceramic; $C_3 = C_4 = 300$ pF; $C_5 = 10$ μF/16 V (electrolytic); electrical length of capacitive branch equals 8.186 cm; electrical length of inductive branch equals 10.49 cm; series line electrical length equals 11.26 cm; and the open-circuit stub length equals 10.49 cm.

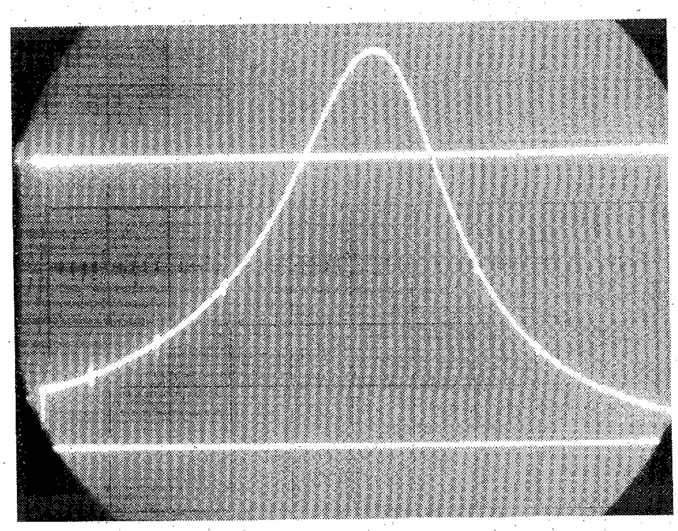

Fig. 7. Display of the single-stage 800-MHz narrow-band amplifier designed using two-port S-parameters. Vertical center line is at 800 MHz. Upper horizontal trace is the 3-dB marker. Frequency markers are 10 MHz apart.

VSWR is given by

$$\text{VSWR}\,|_{g=\infty} = \frac{1 + |\Gamma'|}{1 - |\Gamma'|} = \frac{1 + ||r_s| - \rho_s|}{1 - ||r_s| - \rho_s|} = S.$$

Let $b = ||r_s| - \rho_s|$, and if ν equals maximum tolerable deviation of the VSWR of the system.

$$\text{VSWR}\,|_{\text{stable}} = (1 - 2\nu)S = S'$$

giving

$$|\Gamma| = \frac{2\nu(1 + b) - b}{2 - 2\nu(1 + b)}.$$

By definition, $|\Gamma| = M\,|r_s| - \rho_s| = M \cdot b$

$$M = \frac{2\nu(1 + 1/b) - 1}{2 - 2\nu(1 + b)}$$

$$= \frac{\nu(1 + (||r_s| - \rho_s|)^{-1}) - 0.5}{1 - \nu(1 + ||r_s| - \rho_s|)}.$$

Appendix II

Reflection Detuning Factor

The detuning factor δ_r is defined as the fractional change in the two-port input reflection coefficient with respect to the fractional change in the output load reflection coefficient, i.e.,

$$\delta_{r2} = \left(\frac{\partial\Gamma_1}{\Gamma_1}\right) \Big/ \left(\frac{\partial r_2}{r_2}\right).$$

But

$$\Gamma_1 = S_{11} - \frac{S_{12}S_{21}}{S_{22} - (1/r_2)} = \frac{S_{11} - r_2\Delta}{1 - r_2S_{22}}$$

$$\frac{\partial\Gamma_1}{\partial r_2} = \frac{S_{12}S_{21}}{[S_{22} - (1/r_2)]^2} \cdot \frac{1}{r_2^2}$$

$$\delta_{r2} = \frac{S_{12}S_{21}}{(1 - r_2S_{22})^2} \cdot \frac{r_2/(S_{11} - r_2\Delta)}{1 - r_2S_{22}}$$

$$\delta_{r2} = \frac{r_2S_{12}S_{21}}{(1 - r_2S_{22})(S_{11} - r_2\Delta)}.$$

Hence for a specified value of δ_{r2}

$$S_{11} - r_2\Delta - r_2S_{11}S_{22} + r_2^2\Delta S_{22} - \frac{r_2}{\delta_{r2}} S_{12}S_{21} = 0$$

$$|r_2| = \left| \left(\frac{\Delta + S_{11}S_{22} + (S_{12}S_{21})/\delta_{r2}}{2 . S_{22} . \Delta}\right) \pm \left| \left(\frac{\Delta + S_{11}S_{22} + (S_{12}S_{21})/\delta_{r2}}{2 . S_{22} . \Delta}\right)^2 - \left(\frac{S_{11}}{S_{22}\Delta}\right) \right|^{1/2} \right|.$$

This is the limiting value of the load reflection coefficient.

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Equivalent Circuits of Microstrip Impedance Discontinuities and Launchers

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***Abstract*—Experimental results obtained indicate that an excess phase shift is the most pronounced high-frequency parasitic effect resulting from a microstrip quarter-wave transformer impedance discontinuity. An empirically derived design-oriented model describing the dominant parasitic reactances associated with a microstrip impedance discontinuity at *X*-band frequencies is described. A description is also given of the dominant parasitic reactances associated with a number of commercially available coaxial-to-microstrip launchers.**

I. Introduction

A number of authors [1]–[3] have considered the parasitics associated with impedance discontinuities (such as quarter-wave transformers) in stripline transmission lines. However, very little has been reported concerning impedance discontinuities in microstrip transmission lines [4]. Experimental results obtained by the present authors indicate that an excess phase shift is the most pronounced high-frequency parasitic effect resulting from impedance discontinuities such as in quarter-wave transformers. An empirically derived design-oriented model describing the dominant parasitic reactances associated with a microstrip impedance discontinuity at *X*-band frequencies is described in this short paper. The model is based on the stripline impedance discontinuity analysis reported by Altschuler and Oliner [1], the effective linewidth analysis reported by Leighton and Milnes [4], and the open-circuited transmission line analyses by Altschuler and Oliner [1] and Jain *et al.* [5]. It is shown that the model is valid for characteristic impedance values ranging from 10 to 130 Ω. A description is also given of the dominant parasitic reactances associated with a number of commercially available coaxial-to-microstrip launchers in high VSWR applications (VSWR > 2). The technique used to characterize the launchers is similar to that reported by Weissfloch [6], except that the equivalent-circuit form and parameter values are determined using a simple graphical technique.

II. Experimental Techniques and Launcher Equivalent Circuits

The experimental data reported in this short paper were obtained using structures of the form shown in Fig. 1 and the HP 8410 network analyzer. Published ϵ_{eff} models [7], [8] and experimental resonant ring techniques [9] were used to accurately determine the effective electrical lengths of the various sections of the transformer structures. Traveling microscope measurements of a tolerance of

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